Reading and Writing Strategies for Mathematics

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Abstract
An important aspect of teaching and learning mathematics includes incorporating strategies that target improvement in problem-solving skills. Regardless of the math level being taught, instructors must enable students to approach processes to determine solutions and improve their math skills. Although story problems provide ample opportunities to address innate reasoning and share ideas, unfortunately, when students encounter word problems they often react with anxiety (Swanson, 2010). In addition, math instructors may perceive it as challenging to integrate reading and writing into math lessons. Fortunately, we have many examples of strategies that address varying levels and include English language learners, students with disabilities, and students struggling with math and reading comprehension. Some strategies incorporate visual representation (Ives & Hoy, 2003), and others target basic writing without revisions as well as higher level writing with revision (Wilcox & Monroe, 2011). This paper discusses strategies to provide diverse learners opportunities to be active participants in developing both organizational and reading skills, and reasoning and writing tools. In particular, awareness of students' sensibilities and possible deficiencies in math or language skills are an important aspect of engaging/empowering students to make connections with math and life skills.

Introduction

Regardless of the math level being taught, students often react anxiously or negatively to story problems (Swanson, 2010). When students encounter word problems, they either claim they have never been able to understand story problems or state that they do not know where to begin. This may be a result of lack of knowledge regarding basic definitions and translation skills, issues with reading approaches for organizing data to successfully solve math problems, or common roadblocks that require special attention. These roadblocks do not just exist for English language learners or students with disabilities, but are equally present for native speakers who may have reading issues or math content area literacy issues.

The most common roadblock is student perception that they are math phobic and any failure or frustration from the past crops up at the beginning of discussion on story problems. By being sensitive to possible fears and also considering diverse learning styles and strengths, concepts can be introduced without trivializing the process or insulting the learners' sensibilities. To do so, consider three major phases of reading: pre-, through-, and post-reading, also known as into, through, and beyond (Manzo, Manzo & Thomas, 2005). Incorporate the math topic by connecting the examples to student interests and co-constructing details by interactively analyzing required components for each of these phases.

Pre-Reading phase

Instructor awareness should begin by determining the teaching entry point. What prior knowledge exists, and what gaps and strengths are evident? This phase of introducing mathematical topics begins with probing knowledge of key words for translation to help students construct meaning and gain comprehension. If learners are struggling with basic decoding and comprehension of critical math translation statements, this implies the reading issues will be frustrating. The goal is to eliminate frustrations and work at instructional levels that will guide students until they can independently work on the application problems. Some textbooks take a bottom-up approach by beginning with lower-level definitions and sentence models for translation to develop higher-level understanding. Most texts have chapters that "build up" to word problems after reviewing operations and topics (potentially further alienating students from applications and word problems and possibly building up fears!). An alternate approach is to reverse this technique and begin the introduction to the math topics by illustrating a story problem at the start. Make it clear that this is the ***goal,*** and that the class will work on the details as a group. Be straightforward in addressing student concerns and remind the class that it is normal and acceptable to be unfamiliar with how to approach the problem. Directly state that even if students already know the solution, the purpose of the lesson is to determine procedures and helpful techniques that will help them when they don't know where to begin. Restate that for those who don't know where to begin, at the end of the lesson they will have tools that will enable them to have a starting point for any story problem.

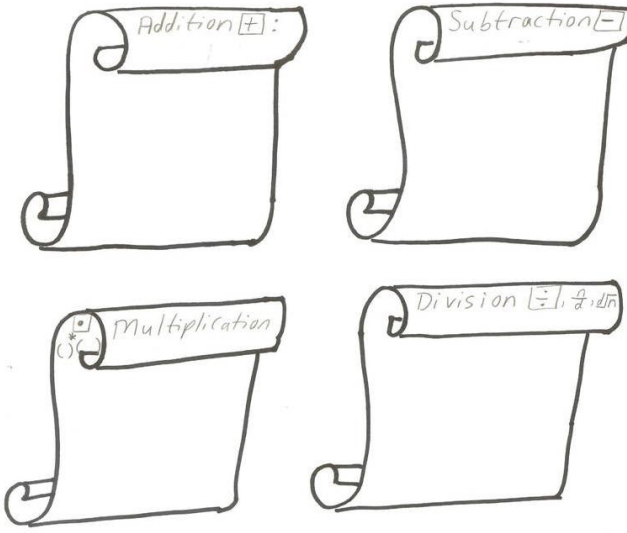
By starting with a story problem, instructors can use an interactive/intervention approach that combines a top-down model to address the general concept and then applies direct instruction to isolate

any skills that need to be built up. The classroom discussion may now focus on context clues and promote interactive analysis. The interactive model of content area literacy in the pre-reading phase targets schema activation. Strategies for this discussion phase include visual prompts that elicit information that is already known by students. The visual representation diagrams are called graphic organizers and successfully assist a diversity of students (Ives & Hoy, 2003). If students are struggling with organizing information, the graphic organizers provide teaching opportunities to generate questions regarding the purpose of the exercise as well as determine the level of prior knowledge.

Besides using graphic organizers as a strategy to address varying levels of math content area literacy, another strategy is to interactively co-construct sentences with the students. For example, the author has used current events articles. One such article discussed the soaring price of gold while working conditions continued to result in serious injuries. Such an article contains information students are aware of and at the same time provides information on human issues and concerns. News articles can be used to elicit questions that may arise regarding the information printed. Students can co-construct the items of interest, like how much has the price of gold increased from one month to another, or how many injuries were reported each month or even how salaries changed from a prior year to the current year. Instructors should be prepared with probing questions that guide students in constructing the potentially important data points to boost their confidence in determining clarifying questions. Help students become aware of extraneous information that occurs naturally in life and inject common key words that translate into mathematical operations, like double, triple, increased by, etc. An effective strategy is to inject the key words in a “think aloud” manner to encourage conjecture.

At this point, visual prompts such as translation buckets or scrolls can be introduced to provide scaffolded opportunities for students to practice translation of sentences to math expressions or equations. The graphic organizer handout in Figure 1 is basic and provides headers and space to write information. The instructor guides the students by asking them to work individually to write alternate words to express the same meaning, and then ask them to pair-share.

Figure 1.



It is best to model one example. For instance, under addition, write the operation symbol, and then list words that mean addition, like sum. Ask students to create a sentence with sum, and then translate this into a math expression or equation as applicable. Ask them what words or translations become tricky to determine stick areas. This is particularly helpful to clear up words like "difference" which indicate the difference between two numbers translates to subtraction, and emphasize the importance of order; words like "of" which at times translates to multiplication, or else is a grammatical part of the sentence. It is important to respect this phase as non-trivial and challenging for some students. Finally, as a group, review all words that they listed and compile a thorough list within these diagrams.

Relying on collaborative strategies provides the biggest foundation for developing conversations and initiating student reflection and consideration of math and/or reading tools they may need. At this point a list of questions and concerns can be gathered for group discussion and clarification.

Another popular graphic organizer for determining teaching entry point and definitions knowledge base is a "KWL" table that discusses what students know, want to know, and have learned. The author inserts a prompt column and integrates the discussion as a collaborative activity (refer to Table 1).

Table 1: KWL Graphic Organizer

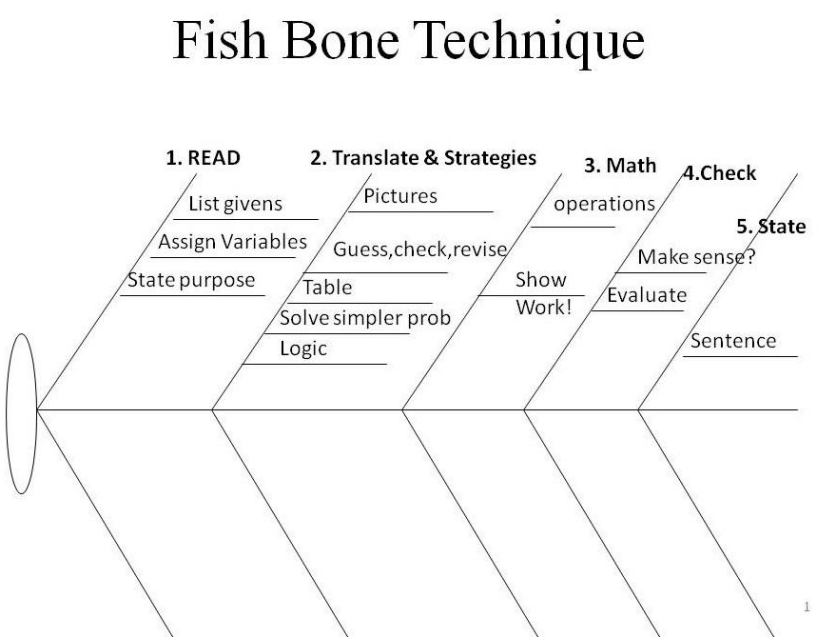
Topic: Geometry questions	K: What do you think you know?	W: What do you want to learn?	L: What have you learned
Perimeter of a rectangle	Example: p = 2l + 2w	How to label the diagram	If you wrote the correct formula, just check this off. If not, rewrite the correct item.

This activity is effective in stimulating student participation and at the end of this "pre-reading" phase; all students should be at the same position to continue learning. Additional helpful aides such as crossword puzzles can be provided so that students can continue review on their own in a task that is often seen as fun and relaxing. With some practice, this phase should only take a few minutes for introduction, a few minutes for individual work, and perhaps several minutes for group interactive discussion, time which arguably is important to providing context and purpose to validity of discussion and critically is important for leveling the "playing" field.

Through-Reading Phase

Once students have familiarized themselves with critical words for determining math operations, a continuation of the pre-reading phase is to organize the information to carry out the operations. A graphic organizer such as the fishbone technique allows everyone to recap the possible translations of the information given, but also to refocus on the purpose of the story problem (refer to Figure 2).

Figure 2.



What questions are we answering? What are the main ideas again? Modeling is most critical at this phase – model how to read, how to use meta-cognitive strategies to see if you are on track, and how to fix any ongoing misconceptions. Allow for a major pause to permit student silent reading and thinking. During their "silent thought" process, request that students either fill out the fishbone diagram, or provide a handout with incomplete sentences to elicit student interaction to complete the information and address the questions in the chosen story problem. To monitor reading comprehension and math procedural requirements, writing is critical.

This is the phase where instructors can provide incomplete sentences (closure technique) to elicit student interaction to complete the information and address the questions in the chosen story problem. The writing is thus brief and to the point. Provide consistent prompts to help students repeat procedures and practice the essential tools for knowing where to begin with word problems. Prompts include:

1) List the given information (Be specific; write a sentence as well as the number for each item given).

2) List the unknowns, assign variables, but again write a sentence describing the unknown.
3) List the questions being asked.
4) State possible math expressions or equations.

Remind students that if they are unsure, they should list the key words that can translate to math operations (addition, subtraction, multiplication, etc.) Encourage self-monitoring at this stage, prompting if they have questions or to list what they don't understand. The author includes collaborative discussion, which culminates in mini-presentations so that students do not feel isolated. As part of the wrap-up, the groups are encouraged to list positives such as issues they addressed and progress points. A positive attitude is maintained even when translations or procedures are wrong, and discussion of why the steps are incorrect is viewed as an opportunity to prevent future errors.

Interactively discuss problem-solving strategies like using simpler numbers for variables to verify the operation, or drawing pictures or graphs to see if the operations used make sense and help predict the results. This is where math instructors spend the most time, but to ensure transfer of skills, students must be given opportunities to have guided practice. Monitor their written graphic organizer or handout to determine which areas are still causing confusion – are they "stuck" on the translation, or on the procedure required? Are they able to describe the given information? Point out the area requiring attention so that the student reconsiders another option for translation or reviews the procedures. This aids in teaching them to fix-up their own process and note-taking prior to addressing the answer.

Take this opportunity to reinforce student interests. Model creation of basic story problems which illustrate math operations and have students verbally create their own unique sentence associated with the math operations. As a group, discuss whether the sentence provides sufficient information and continue to promote rewording of the problems. Mistakes happen, so encourage catching these errors to animate conversation. Be mindful of different learning styles and provide verbal, written, spatial and visual cues. Encourage students to be supportive of each other and provide opportunities for them to practice both organizing information skills as well as carrying out math processes skills. Creating math menus of topics (such as student activities like hiking, diving, football, or money or money in bank accounts, loans, credit cards) often helps to alter standard textbook examples to examples that are pertinent to students. It is the author's

opinion that this phase is the most class intensive in terms of time, but well worth the expenditure. Students must be able to identify math strategies and learn self-questioning and creative skills. The strategies need to be reinforced in multiple class lessons, and not just as one section in one chapter.

Post-Reading Phase

To recap math processes and ensure transfer of skills, create assignments that target basic writing with or without revisions (Wilcox & Monroe, 2011). Clearly, some writing has already occurred at the previous phases, but at this final stage students are expected to organize what they have learned and formulate valid conclusions. In the spirit of keeping things simple, at the very least request that students write a complete sentence addressing the questions posed in given story problems. Although story problems can be challenging for students, research indicates more student success in solving simple story problems (Koedinger & Nathan, 2004). If the story problems are simple, this allows for ample opportunities for a brief writing of conclusions. Keeping this short will be appreciated by both student and instructor, and since the main target is higher-level thinking and math procedures, it will also encourage brief reflection. Sometimes students become nervous with this request or simply ignore the request. This issue will clear up if there is some direct discussion stating that points will not be deducted for grammar or spelling, but only awarded for completing the sentence.

If you wish to pursue alternative assessment projects, then math journals and portfolios are perfect examples of the post-reading phase that advances student understanding and permits them to integrate math processes and math applications in story problems to improve their communication skills. The author has included a creative writing portion in math journals that require students to formulate a story problem associated with chapter content and show all work for solving the problem. Additional questions and prompts for students may address homework and/or exam correction analysis. Provide some guided options as to the problem source (examples include “careless mistake,” “did not understand the problem,” and so forth).

This phase can be broken out into smaller writing prompts or projects that are due at the end of class or as a homework assignment. An example writing prompt is, “Your friend missed class today and asks you to please restate how to…” with a fill-in for the particular math topic you wish

to have students address ("Tips on…," 2012). Reflection questions can be given after quizzes or exams to encourage student self-reflection and analysis.

Conclusion

This paper has focused on integrating three phases of reading to improve math literacy and help students with problem-solving skills for story problems. At the pre-reading phase, student background is reviewed to correct any misunderstandings and review key concepts required for successful communication of ideas. Strategies for assisting diverse learner levels include:

1) Introduce relevant examples that allow students to connect and relate to the story application and permits for casual thinking aloud;
2) Use of visuals such as graphic organizers which address organizing information and activating relevant schema; and
3) Promote collaborative discussion to engage the student as a reader analyzing and constructing meaning.

At the through-reading phase, instructors:

1) Model successful reading and math strategies;
2) Model self-questioning and auto/self-correction; and
3) Provide handouts or graphic organizer examples with prompts that encourage writing what is understood as well as what issues exist.

This phase is time-consuming and requires patience and enthusiastic support. Remember to keep it pertinent to students so that they value the thinking process. And finally, the third phase is the post-reading phase. This phase allows for individual ownership as well as retaining the information. A variety of post-reading assignments include:

1) Simple sentence conclusion associated with story problem questions,
2) Writing prompts/projects as in-class or homework assignments that promote reflection/retention of procedures,
3) Math journals or portfolios that include self-reflection and examining problem solving strategies.

Integrating reading strategies with math strategies helps to address diversity and perceived deficiencies. Engaging students in creating pertinent links to applications and story problems with life skills improves communication and empowers students to self-reflect and hopefully replace math anxiety with engagement and interest.

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Biographical Information

Natalia Darling is an Assistant Professor of Mathematics at the University of Cincinnati, Blue Ash College (UCBA.) She teaches courses that include Preparatory Mathematics, College Algebra and Calculus. Professor Darling received her B.S. in Electrical Engineering from Polytechnic University in Brooklyn, NY, a Masters of Engineering from Cornell University in Ithaca, NY, and a Masters in Secondary Education from Xavier University in Cincinnati, OH. She has K-12 certification specializing in Spanish. Her career experience covers areas in the telephony research and in technology support and

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